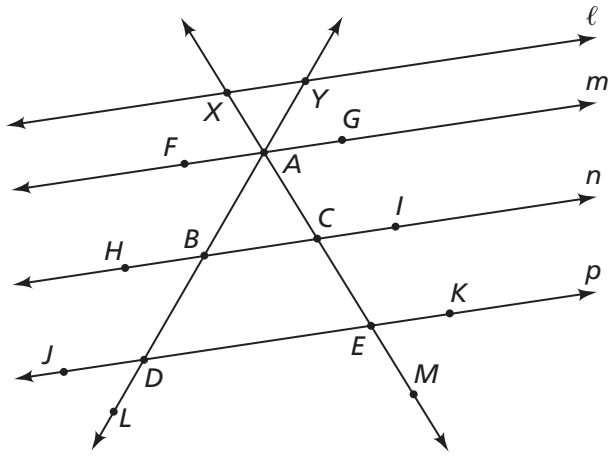
Topic 6: Parallel and Perpendicular

for use after ***The Shapes of Algebra*** **Investigation 2**

In the diagram below, lines ℓ, m, n, and p are parallel lines. The other two lines are **transversals.** Angles ACI and CEK are in corresponding positions at the vertices C and E. Each is in the "top-right" or "north-east" position at their vertices. Because of their corresponding positions, they are called **corresponding angles.** Corresponding angles are congruent to each other if they are formed by a transversal intersecting parallel lines. Angles BAC and XAY are **vertical angles.** Vertical angles are always congruent to each other.



When parallel lines are cut by non-parallel transversals, similar triangles are formed. In this figure, triangle ABC is similar to triangle ADE. Corresponding sides of these similar triangles form equal ratios. For example:

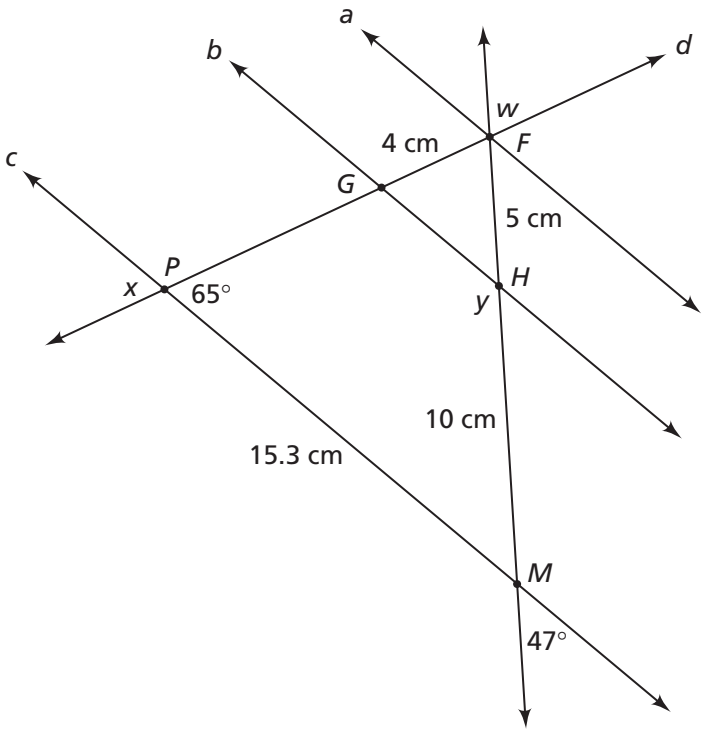
$$\frac{\text{length of } AC}{\text{length of } \overline{AE}} = \frac{\text{length of } AB}{\text{length of } \overline{AD}}$$

Exercises

For Exercises 1–6, use the diagram above.

1. **a.** List five other pairs of vertical angles in the diagram.
   b. List five other pairs of corresponding angles in the diagram.
2. What other segments form equal ratios? Explain.
3. What angles are congruent to $\angle CEK$? Explain.
4. What angles are congruent to $\angle BDJ$? Explain.
5. Why are $\angle EDL$ and $\angle BDJ$ congruent?
6. What other triangle is similar to triangle ABC? Explain.

For Exercises 7–13, use the diagram below. Lines *a, b*, and *c* are parallel.



7. What are the measures of $\angle PMH$ and $\angle x$?

8. What are the measures of $\angle FGH$ and $\angle GHF$?

9. What is the measure of $\angle PGH$?

10. What is the measure of $\angle y$?

11. What is the length of $\overline{GP}$?

12. What is the measure of $\angle GFH$ and $\angle w$?

13. What is the length of $\overline{GH}$?

Remember that lines are parallel if their slopes are equal and lines are perpendicular if their slopes are negative reciprocals of each other.

Sample Are the lines $y = 2x + 8$ and $3 = 2x - y$ parallel?

Rewrite the second equation.

$$\begin{aligned} 3 &= 2x - y \\ y + 3 &= 2x - y + y \\ y + 3 &= 2x \\ y &= 2x - 3 \end{aligned}$$

The slope of this line is 2, which is also the slope of the first line. The slopes are equal, so the lines are parallel.

Sample Are the lines $y = 2x + 8$ and $7 = \frac{1}{3}x - y$ perpendicular?

Rewrite the second equation.

$$\begin{aligned} 7 &= \tfrac{1}{3}x - y \\ y + 7 &= \tfrac{1}{3}x - y + y \\ y + 7 &= \tfrac{1}{3}x \\ y &= \tfrac{1}{3}x - 7 \end{aligned}$$

The slope of this line is $\frac{1}{3}$. The slope of the first line is 2. The slopes are not negative reciprocals of each other, so the lines are not perpendicular.

Determine whether each pair of lines is parallel, perpendicular, or neither.

14. $y = 5x - 7$
$y + 5x = 12$

15. $y = x - 0.5$
$y + x = 0.25$

16. $y = \frac{1}{2}x - \frac{3}{4}$
$y - \frac{1}{2}x = \frac{5}{6}$

17. $2y = 6x - 72$
$y - 3x = 15$

18. $y + x = 12$
$y - x = 12$

19. $5x - y = 12$
$5y + x = 35$

At a Glance

Topic 6: Parallel and Perpendicular

PACING 1 day

Mathematical Goals

- Investigate parallel and perpendicular algebraically and geometrically
- Apply properties of angle pairs formed by parallel lines and transversals
- Understand properties of the ratio of segments when parallel lines are cut by transversals

Guided Instruction

Materials
- Labsheet 6.1

Vocabulary
- transversals
- corresponding angles
- vertical angles

The terminology of corresponding angles and vertical angles may be new to students. Corresponding angles are congruent only if they are formed by a transversal intersecting parallel lines. If they are formed by a transversal intersecting non-parallel lines, then the corresponding angles are *not* cougruent. Vertical angles are always congruent. Students use these facts to identify pairs of congruent angles. Before students begin to solve the exercises, you may want to help students remember what they know about proportional reasoning and similar triangles.

Because angles are named with triads of letters, many of the angles in the figure on the first page have multiple names. The letter representing the vertex of the angle will be the same for each of the names. For example, $\angle GAC$ can also be named $\angle GAE$ and $\angle GAM$. The answers given below use only one name for each angle. Students may use different names for the angles.

Students may have some difficulty identifying five pairs of vertical and corresponding angles in Exercise 1. You may want to ask students to work with a partner or in small groups to find all of these pairs of angles. One strategy that may help students find corresponding angles is to imagine that one of the transversals is removed from the figure. Students might actually cover up one of the transversals with a finger.

For the topic introduction, ask:

- *What does perpendicular mean?*
- *What does parallel mean?*
- *Can you name angles* ACI *or* CEK *in more than one way?*
- *What does it mean for ratios to be equal?*
- *Suppose that one of the transversals is removed. Is it easier to find pairs of corresponding angles?*

For the section after Exercise 13, ask:

- *What are negative reciprocals?*
- *What is the slope of a line?*

Assignment Guide for Topic 6

Core 1–19

Answers to Topic 6

Exercises

1. **a.** Any five of the following will do:
   $\angle XAY$ and $\angle BAC$, $\angle XAF$ and $\angle GAC$, $\angle YAG$ and $\angle FAB$, $\angle ACB$ and $\angle ICE$, $\angle ACI$ and $\angle BCE$, $\angle ABC$ and $\angle HBD$, $\angle ABH$ and $\angle CBD$, $\angle CEK$ and $\angle DEM$, $\angle KEM$ and $\angle CED$, $\angle BDE$ and $\angle JDL$, $\angle BDJ$ and $\angle LDE$

   b. Any five of the following will do:
   $\angle XAG$ and $\angle ACI$, $\angle XAG$ and $\angle CEK$, $\angle ACI$ and $\angle CEK$

   $\angle YXA$ and $\angle GAC$, $\angle YXA$ and $\angle ICE$, $\angle YXA$ and $\angle KEM$, $\angle GAC$ and $\angle ICE$, $\angle GAC$ and $\angle KEM$, $\angle ICE$ and $\angle KEM$

   $\angle XAF$ and $\angle ACB$, $\angle XAF$ and $\angle CED$, $\angle ACB$ and $\angle CED$

   $\angle FAC$ and $\angle BCE$, $\angle FAC$ and $\angle DEM$, $\angle BCE$ and $\angle DEM$

   $\angle YAG$ and $\angle ABC$, $\angle YAG$ and $\angle BDE$, $\angle ABC$ and $\angle BDE$

   $\angle GAB$ and $\angle CBD$, $\angle GAB$ and $\angle EDL$, $\angle CBD$ and $\angle GAB$

   $\angle XYA$ and $\angle FAB$, $\angle XYA$ and $\angle HBD$, $\angle XYA$ and $\angle JDL$, $\angle FAB$ and $\angle HBD$, $\angle FAB$ and $\angle JDL$, $\angle HBD$ and $\angle JDL$

   $\angle YAF$ and $\angle ABH$, $\angle YAF$ and $\angle BDJ$, $\angle ABH$ and $\angle BDJ$

2. Proportions can be written in multiple ways, so the answers below are samples.
   $\overline{AX} : \overline{XY} = \overline{AC} : \overline{CB} = \overline{AE} : \overline{ED}$
   $\overline{AY} : \overline{XY} = \overline{AB} : \overline{BC} = \overline{AD} : \overline{DE}$
   $\overline{AX} : \overline{AY} = \overline{AC} : \overline{AB} = \overline{AE} : \overline{AD}$
   Corresponding parts of similar triangles are congruent. The transversals create three similar triangles: triangle AXY, triangle ACB, and triangle AED.

3. $\angle CEK$ is congruent to $\angle ACI$ and $\angle XAG$ (corresponding angles), and $\angle DEM$, $\angle BCE$, and $\angle FAC$ (vertical angles for the first three angles).

4. $\angle BDJ$ is congruent to $\angle ABH$ and $\angle YAF$ (corresponding angles), and $\angle EDL$, $\angle CBD$, and $\angle GAB$ (vertical angles for the first three angles).

5. They are vertical angles.

6. Triangle AED is similar, because the angles are congruent. Triangle AYX is similar, because the angles are congruent.

7. $\angle PMH$ is 47°, $\angle x$ is 65°.

8. $\angle FGH$ is 65°, $\angle GHF$ is 47°.

9. $\angle PGH$ is 115°.

10. $\angle y$ is 133°.

11. $\overline{GP}$ is 8 cm. **Note:** $\triangle FGH \sim \triangle FPM$ and the scale factor is 3.

12. $\angle GFH$ is 68°, $\angle w$ is 68°.

13. 5.1 cm

14. Neither; the slopes are $+5$ and -5.

15. Perpendicular; the slopes are $+1$ and -1 (negative reciprocals).

16. Parallel; the slopes are $\frac{1}{2}$ and $\frac{1}{2}$ (equal).

17. Parallel; the slopes are 3 and 3 (equal).

18. Perpendicular; the slopes are 1 and -1 (negative reciprocals).

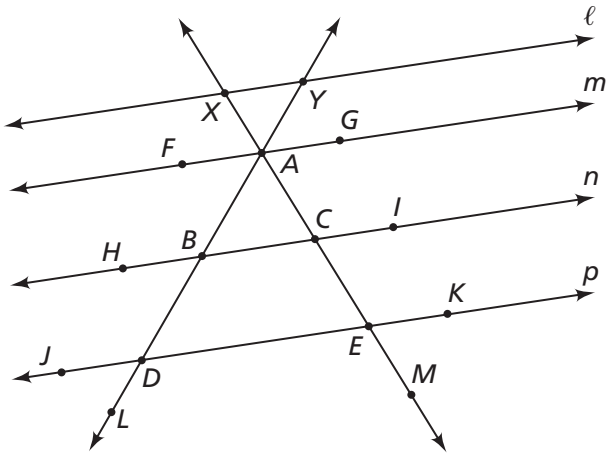
19. Perpendicular; the slopes are 5 and $-\frac{1}{5}$ (negative reciprocals).

Name _____ Date _____ Class _____

Labsheet 6.1

Topic 3

Introduction



Exercises 8–14